This homework covers the contents of *Loading Effects* of sensor devices, *Resistive sensors*, *Temperature sensors*.

1. Potentiometers are frequently used to measure linear or angular displacements; it is expected that the output voltage of the sensor circuit is proportional to the fractional displacement.

1). Suppose the sensor circuit has a supply voltage $V_s$ and load resistor $R_L$, draw the Thevenin equivalent circuit of this sensor system 5%

2). Show that the relationship between output voltage $V_L$ and input displacement fraction $x$ is nonlinear. 5%

4). Suppose the load resistance $R_L$ is $10\, \text{K}\Omega$ and we require that the maximum nonlinearity of this sensor system is less than 3% find a suitable potentiometer resistance. 10%

5). If the rated power dissipation is 0.1Watt, and the total angular displacement range of the sensor is $\left(11/6\pi \right)$ (or $330^\circ$), find an expression of the sensitivity of this sensor. 10%.

Ans: 

1). 

\[
x = \frac{d}{d_T}
\]
2.)

\[
\frac{1}{R_{th}} = \frac{1}{R_p x} + \frac{1}{R_p (1-x)}
\]

\[
R_{th} = \frac{R_p (1-x) R_p x}{R_p (x + 1-x)} = R_p (1-x)
\]

\[
V_L = \frac{R_L}{R_L + R + h} = V_s \times \frac{R_L}{R_p (1-x) + R_L} = V_s \times \frac{1}{R_L (1-x) + 1}
\]

\[\therefore\text{output voltage } V_L \text{ and input fraction } x \text{ is nonlinear.}\]

4.)

\[N(x) = V_s \left\{ \frac{x^2 (1-x) \left( \frac{R_p}{R_L} \right)}{1 + \left( \frac{R_p}{R_L} \right) x (1-x)} \right\} \]

if \( \left( \frac{R_p}{R_L} \right) \ll 1 \)

\[N(x) \text{ has a maximum value of} \]

\[
\hat{N} = \left( \frac{4}{27} \frac{V_s \left( \frac{R_p}{R_L} \right)}{V_s} \right) \times 100\%
\]

\[\Rightarrow 3 \geq 400 \left( \frac{R_p}{R_L} \right) \]

\[\Rightarrow 3 \geq 400 \left( \frac{R_p}{10K} \right) \]

\[\Rightarrow R_p \leq 2025\Omega \]

選用 \( R_p = 2k\Omega \) 電阻

5.)

以 \( R_p = 2k\Omega \) 為例:

\[\frac{dV_L}{dx} \approx V_s \quad \frac{V_s^2}{R_p} \leq 0.1W \Rightarrow V_s \leq \sqrt{0.1 \times 2000} = 14.14V\]

故 sensitivity = \( 14.14/330 = 42.85 \text{ mv/度} \)
2. Give an example to show that a buffer amplifier can improve the loading effect of a sensor device.10%

Ans:

已知一 pH glass electrode 其 $E_{Th}=59\text{pHmV}$, $Z_{Th}=10^9\Omega$, 一 indicator 其 $Z_L=R_L=10^4\Omega$, 另有一 buffer amplifier 其 $Z_{in}=10^{12}\Omega$, $Z_{out}=10\Omega$。

(1) pH glass electrode 與 sensor system 之 indicator 相連（如下圖中所示）:

![Diagram 1](image1)

由公式 [5.2] 可得

$$pH_M = V_L \cdot \frac{1}{59} = E_{Th} \cdot \frac{Z_L}{Z_{Th} + Z_L} \cdot \frac{1}{59} = 59 \cdot \left( \frac{10^4}{10^4 + 10^9} \right) \cdot \frac{1}{59} \approx 10^{-5}\text{pH}$$

(2) pH glass electrode 與 sensor system 之 indicator 間加上一 buffer amplifier (unity gain)（如下圖中所示）:

![Diagram 2](image2)

$$pH_M = V_L \cdot \frac{1}{59} = 59 \cdot \frac{10^{12}}{10^{12} + 10^9} \cdot \frac{10^4}{10^4 + 10} \cdot \frac{1}{59} = 0.998\text{pH}$$

由(1)及(2)中的討論可得知，(1) 式中的 loading error ≈ -1pH，而(2) 中所得的 loading error 為 -0.002pH，由此可知，透過一 buffer amplifier 可大幅改善系統的 loading effect。
3. An electronic differential transmitter gives a current output of 4 to 20 mA linearly related to a differential pressure input of 0 to 10$^4$ Pa. The Norton impedance of the transmitter is 10$^5$Ω. The transmitter is connected to an indicator of impedance 250Ω via a cable of total resistance 500Ω. The indicator gives a reading between 0 and 10$^4$ Pa for an input voltage between 1 and 5V. Calculate the system measurement error, due to loading, for an input pressure of 5x10$^3$ Pa.  20%

Ans:

1. 求得輸出電流:
   由內插法得知
   \[
   \frac{10^4 - 5 \times 10^3}{10^4 - 0} = \frac{20 - i_N}{20 - 4} \implies i_N = 12mA
   \]
   利用課本 pp. 73 之[5.15]式:
   \[V_R = i_NR_N, \quad \frac{R_N}{R_N + R_C + R_R}\]
   \[\Rightarrow VR = 0.012 \times 250 \times \frac{10^5}{250 + 500 + 10^5} = 2.9777V\]

2. 求得輸出壓:
   再透過內插法得:
   \[
   \frac{5 - 2.9777}{5 - 1} = \frac{10^4 - P_{\text{measure}}}{10^4 - 0} \implies P_{\text{measure}} = 4944.25Pa
   \]
   故 system measure error = $P_{\text{measure}}- P_{\text{input}} = 4944.25-5000 = -55.75Pa$

4. A platinum resistance sensor is to be used to measure temperatures between and 200$^\circ$ C. Given that the resistance $R_T$Ω at $T^\circ$ C is given by $R_T=R_0(1+\alpha T+\beta T^2)$ and $R_0=100.0$, $R_{100}=138.50$, $R_{200}=175.83$Ω calculate: 15%

1) The value of $\alpha$ and $\beta$

2) The non-linearity at 100$^\circ$ C as a percentage of full-scale deflection.

Ans:

(1) \[
\begin{align*}
138.50 &= 100(1 + 100\alpha + 10000\beta) \\
175.83 &= 100(1 + 400\alpha + 40000\beta)
\end{align*}
\]
   \[\implies \alpha = 3.91 \times 10^{-3} \left(\text{s}/\text{C}^{-1}\right), \quad \beta = -5.85 \times 10^{-7} \left(\text{s}/\text{C}^{-2}\right)\]

(2) $K = \frac{175.83 - 100}{200 - 0} = 0.3792$
   故線性曲線 $R_{TL} = 0.3792T + 100$
   $R_{100} = 0.3792 \times 100 + 100 = 137.92$
   non-linearity = $\frac{138.50 - 137.92}{175.83 - 100} \times 100\% = +0.76\%$
5. When using bonded strain gage to measure force,

1) Give the favorable features and limiting factors of this type of sensor
2) Show that a deflection bridge circuit can convert the measured strain to a voltage.
3) Show that a three-wire connection in deflection bridge is required for precision measurement when only using one single strain gage.
4) Estimate the possible voltage level when the supply voltage is 10 Volts in the bridge circuit. (Use the strain gage example given in the text book pp.142.)

20%

Ans :

(1)

- **Favorable Factors**:
  - Small size and very low mass
  - Fully bonded to basic spring structure (shock-resistance)
  - Excellent linearity over wide range of strain
  - Highly stable with time
  - Relatively low in cost
  - Circuit output is a resistance change

- **Limiting Factors**:
  - Thermal degradation
  - Output signal is relative low
  - Careful installation procedure required
  - Moisture effect

(2)

\[
V_{out} = \frac{R}{R + R} E - \frac{R}{R + (R + \Delta R)} E = \frac{1}{2} E - \frac{RE}{2R + \Delta R}
\]

\[
= \frac{(2R + \Delta R)E - 2RE}{2(2R + \Delta R)} = E \frac{\Delta R}{4R + 2\Delta R}
\]

Since \(4R \gg \Delta R\) \(\Rightarrow\) \(V_{out} = \frac{\Delta RE}{4R} = \frac{E}{4} (G.F.)\varepsilon\)

Where G.F. : gage factor
\(\varepsilon\) : strain
E : supply voltage
由上圖可知，應用時往往 Strain Gage 與 deflection bridge 間相距一段不短的距離，故需利用一長導線將兩部份連接，但於精密的量測時，會發生導線的阻值與 Strain gage 所得的△R 相較下太大而不可忽略的情形發生。

我們將發現造成輸出的電壓的改變並非只由於形變所造成，甚至可能因 \( R_{wire} \) 過大而使形變所造成的改變被忽略掉；若改採用右圖中的 three-wire connection 的設計，由於 Amp. 為高阻抗元件，故使得流經第三條導線之電流趨近於零，故所得之輸出電壓受到導線電阻的影響減少，以下式子推導更可說明使用 three-wire connection 後，\( R_{wire} \) 將不影響結果。

由上圖可知，此時所得之輸出電壓變化量為：

\[
V_{out} = \frac{R}{R + R} E - \frac{R + R_{wire}}{(R + R_{wire} + \Delta R) + (R + R_{wire})} E
\]

\[\rightarrow \text{假若我們不考慮} \Delta R \text{效應，上式將變成} \]

\[
V_{out} = \frac{R}{R + R} E - \frac{R + R_{wire}}{2(R + R_{wire})} E
\]

\[= 0 \]

（4）

G.F.：2.0~2.2

maximum tensile strain：+2x10^{-2}

maximum compressive strain：−1x10^{-2}

\[
V_{out,max} = \frac{10}{4} \times 2.2 \times 2 \times 10^{-2} = 0.11(V)
\]

\[
V_{out,min} = \frac{10}{4} \times 2.2 \times (−1) \times 10^{-2} = −0.055(V)
\]
6. Suppose we wish to measure the temperature $T_1 \, ^\circ C$ of a liquid inside a vessel with a iron vs. constantan thermocouple (type J). The measurement junction is inserted in the liquid and the reference junction is outside the vessel, where the temperature is $15 \, ^\circ C$. The measured e.m.f. is 5.3025mV using a voltmeter inserted at the reference junction. The value of $E_{15,0}$ is 0.762mV using the thermocouple table (see the handout given in the classroom). Try to find the liquid temperature.

Ans:

1. 由中間溫度定律

\[
E_{T,0} = E_{T,15} + E_{15,0}
\]

\[
= 5.3025 + 0.762
\]

\[
= 6.0645 \, \text{mV}
\]

2. 根據查表並使用內插法，

\[
T_{114^\circ C} = 6.031 \, \text{mV} \quad T_{115^\circ C} = 6.085 \, \text{mV}
\]

\[
T = 114 + \frac{1 \, ^\circ C}{6.085 - 6.031} \times (6.0645 - 6.031)
\]

\[
= 5.3025 + 0.762
\]

\[
= 114.62 ^\circ C
\]