1. A platinum resistor sensor is to be used to measure temperatures between 0 and 200°C. Given that the resistance $R_T \Omega$ at $T^0C$ is given by $R_T = R_0(1 + \alpha T + \beta T^2)$ and $R_0=100.0, R_{100}=138.50, R_{200}=175.83$, calculate: 15%

1) The value of $\beta$ and $\alpha$
2) The non-linearity at 100°C as a percentage of full-scale deflection.

Ans:

(1) $\begin{align*}
138.50 &= 100(1 + 100\alpha + 10000\beta) \\
175.83 &= 100(1 + 400\alpha + 40000\beta)
\end{align*}$

$\Rightarrow \alpha = 3.91 \times 10^{-3} \left( ^0C^{-1} \right) \quad \beta = -5.85 \times 10^{-7} \left( ^0C^{-2} \right)$

(2) $K = \frac{175.83 - 100}{200 - 0} = 0.3792$

故線性曲線 $R_{TTL} = 0.3792T + 100$

$R_{100} = 0.3792 \times 100 + 100 = 137.92$

non-linearity = $\frac{138.50 - 137.92}{175.83 - 100} \times 100\% = +0.76\%$

2. An iron vs. constantan thermocouple is to used to measure temperature between 0 and 300°C. The e.m.f values are given in Table 8.2(attached). 15%

1) Find the non-linearity at 100°C and 200°C as a percentage of full-scale
2) Between 100 and 300°C the thermocouple e.m.f is given by $E_{T,0} = a_1T + a_2T^2$, calculate $a_1$ and $a_2$
3) The e.m.f is 12500uV relative to a reference junction of 20°C and the corresponding reference junction circuit voltage is 1000uV. Use the result of 2) to estimate the measured junction temperature.
Ans: (1) Ideal straight line: \[ O = \frac{16325}{100} \times I \]

\[ \text{Ideal} E_{100,0} = \frac{16325}{300} \times 100 = 5442 \]

non-linearity at \( 100^\circ C \Rightarrow \frac{5268 - 5442}{16325} \times 100\% = -1.07\% \)

\[ \text{Ideal} E_{200,0} = \frac{16325}{300} \times 200 = 10883 \]

non-linearity at \( 200^\circ C \Rightarrow \frac{10777 - 10883}{16325} \times 100\% = -0.65\% \)

(2) \[
\begin{align*}
E_{100,0} &= 100a_1 + 10000a_2 = 5268 \\
E_{300,0} &= 300a_1 + 90000a_2 = 16325 \\
\end{align*}
\]

\( \Rightarrow a_1 = 51.8 \mu V^\circ C^{-1}, a_2 = 8.68 \times 10^{-3} \mu V^\circ C^{-2} \)

(3) \( E_{r,20} = 12500, E_{20,0} = 1000 \Rightarrow E_{r,0} = 13500 \)

\( 13500 = 51.8T + 8.68 \times 10^{-3} T^2 \Rightarrow T = 248^\circ C \)

3. A piezoelectric crystal, acting as a force sensor, is connected by a short cable of negligible capacitance and resistance to a voltage detector of infinite bandwidth and a purely resistive impedance of 10M, 15%

1) Use the crystal data given below to calculate the system transfer function and to sketch the approximate frequency response characteristics of the system.

2) A time variation of input force is a square wave of period 10ms. Explain why the above system is unsuitable for this application.

3) A charge amplifier with feedback capacitance \( C_F = 1000 \text{pF} \) and feedback resistance \( R_F = 100 \text{M}\Omega \) is incorporated into the system. By sketching the frequency response of the modified system, explain why suitable for part 2).

Crystal data

- Charge sensitivity to force = 2pC N\(^{-1}\)
- Capacitance = 100pF
- Natural frequency = 37kHz
- Damping ratio = 0.01
Ans:

(1)

\[ \tau = R_L(C_N + C_C) = 10 \times 10^6 \times (0 + 100 \times 10^{-12}) = 0.001 \]

\[ \xi = 0.01 \]

\[ \frac{d}{C_N + C_C} = \frac{2 \times 10^{-12}}{100 \times 10^{-12}} = 0.02 \]

\[ G(s) = \frac{\frac{d}{C_N + C_C} \frac{\pi}{\omega_n} \frac{1}{s^2 + \frac{2\xi}{\omega_n} s + 1}}{1 + \frac{0.001s}{s^2 + 4644s + 5.4 \times 10^{10}}} \]

\[ = 0.02 \times \frac{0.001s}{1 + 0.001s} \times \frac{5.4 \times 10^{10}}{s^2 + 4644s + 5.4 \times 10^{10}} \]

\[ \Rightarrow \text{圖(3-1)為原系統之頻率響應圖。} \]

图(3-1) 原系统频率响应图

(2)

由系统的频率响应可以看出此感应器的低频响应并不好，此系统有效的频率输入范围必须大于 \(1/\tau=1000\) rad/s，而我们输入之方波的频率...
\[ f = \frac{1}{0.01} = 100 \text{Hz}, \omega = 628 \text{ rad/s}, \text{因此我們將發現此系統並不適合量測此外力。} \]

\[ G(s) = \frac{d}{C_F} \left( \frac{1}{\omega_n^2 s^2 + \frac{2\xi}{\omega_n} s + 1} \right) = 0.02 \times \frac{5.4 \times 10^{10}}{s^2 + 4644s + 5.4 \times 10^{10}} \]

我们發現當加入 charge amplifier 後，將使得 \( \frac{\xi}{1 + \xi} \) 項消失，以至於在 \( \omega = 0 \rightarrow |G(j\omega)| = 1 \)。

\( \tau = R_p C_F = 10^8 \times 10^{-9} = 0.1 \)

\( \omega = \frac{1}{\tau} = 10 \text{rad/s} \)

因為 628rad/s > 10rad/s，所以我們現在即可量測到此外力。

圖(3-2)為加入 charge amplifier 之系統頻率響應圖
4. Incremental shaft encoders are used in angular position measurement. Given a DC motor with an encoder of 1000 ppr attached on the shaft.

1) Explain how the angular displacement can be measured
2) How a HCTL-2000 Chip can be used in this application?
3) When the motor rotate an angle of 180°, what will be the change of the counter content in HCTL-2000?

Ans:

(1)
When the motor rotates, the index pulse on the encoder will follow the rotation, and the internal optical element will intermittently receive the optical pulse. It will produce a series of square waveforms, and we can determine the angle displacement from the waveform changes of channel A and channel B.

(2)
HCTL-2000 chip's functions:
1. quadrature decoder (1 pulse à 4 counts)
2. Counter (12-bit, 16-bit)
3. Digital interface (to data bus)

➢ To convert 1 pulse to 4 counts:

```
<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
```

(3)
quadrature counts = 4×1000 ppr
= 4000 counts

180° change in counts = \( \frac{4000 \times 180}{360} = 2000 \) counts
5. When using bonded strain gage to measure force,
   1) Give the favorable features and limiting factors of this type of sensor
   2) Show that a deflection bridge circuit can convert the measured strain to a voltage.
   3) Show that a three-wire connection in deflection bridge is required for precision measurement when only using one single strain gage.
   4) Estimate the possible voltage level when the supply voltage is 10 Volts in the bridge circuit. (Use the strain gage example given in the text book pp.142)  

Ans :
   (1)
   • Favorable Factors :
     ➢ Small size and very low mass
     ➢ Fully bonded to basic spring structure(shock-resistance)
     ➢ Excellent linearity over wide range of strain
     ➢ Highly stable with time
     ➢ Relatively low in cost
     ➢ Circuit output is a resistance change

   • Limiting Factors :
     ➢ Thermal degradation
     ➢ Output signal is relative low
     ➢ Careful installation procedure required
     ➢ Moisture effect

   (2)
   \[
   V_{out} = \frac{R}{R + R} E - \frac{R}{R + (R + \Delta R)} E = \frac{1}{2} E - \frac{RE}{2R + \Delta R}
   \]
   \[
   = \frac{(2R + \Delta R)E - 2RE}{2(2R + \Delta R)} = E \frac{\Delta R}{4R + 2\Delta R}
   \]
   Since \( 4R \gg \Delta R \Rightarrow V_{out} = \frac{\Delta RE}{4R} = \frac{E}{4}(G.F.)\varepsilon \)

   Where G.F. : gage factor
   \( \varepsilon \) : strain
   E : supply voltage
由上頁左圖可知，應用時往往 Strain Gage 與 deflection bridge 間相距一段不短的距離，故需利用一長導線將兩部份連接，但於精密的量測時，會發生導線的阻值與 Strain gage 所得的 $\Delta R$ 相較下太大而不可乎略的情形發生，此時所得之輸出電壓變化量為:

$$V_{out} = \frac{R}{R+R} E - \frac{R+R_{wire}}{R+2R_{wire}+(R+\Delta R)} E \quad \text{不考慮} \Delta R$$

$$= \frac{R}{R+R} E - \frac{R+R_{wire}}{2(R+R_{wire})} E$$

$$= 0$$

假若我們不考慮 $\Delta R$ 的效應，上式將變成

$$V_{out} = \frac{R}{R+R} E - \frac{R+R_{wire}}{2(R+R_{wire})} E$$

$$= 0$$

由上面兩個式子，我們將發現造成輸出的電壓的改變並非只由於形變所造成，甚至可能因 $R_{wire}$ 過大而使形變所造成的改變被乎略掉；若改採用右圖中的 three-wire connection 的設計，由於 Amp. 為一高阻抗元件，故使得流經第三條導線之電流趨近於零，故所得之輸出電壓受到導線電阻的影響減少，以上式子推導更可說明使用 three-wire connection 後，$R_{wire}$ 將不影響結果。

(4)

G.F. : 2.0~2.2

maximum tensile strain : $+2 \times 10^{-2}$

maximum compressive strain : $-1 \times 10^{-2}$

$$V_{out,max} = \frac{10}{4} \times 2.2 \times 2 \times 10^{-2} = 0.11(V)$$

$$V_{out,min} = \frac{10}{4} \times 2.2 \times (-1) \times 10^{-2} = -0.055(V)$$
6. Try to give a design using an LVDT to measure the opening of a two-fingered robot gripper.

1) Try to give a description of your design.
2) Discuss how to calibrate your system.

Ans:

(1)

利用右邊的機械臂來帶動 LVDT 的 core，以求得移動所造成的輸出訊號改變。

(2)

用兩手指緊閉至完全張開期間，量取數點兩手指之間的距離與其對應之 LVDT 輸出電壓值：再利用反向測量數點由兩手指完全張開至完全緊閉時，兩手指間的距離與其對應之 LVDT 輸出之電壓值。利用來回所量得數點距離與電壓之關係，以最小平方線性化的方法，求取距離與輸出電壓的關係。

7. Suppose we wish to measure the temperature $T_1$°C of a liquid inside a vessel with a chromel v. alumel thermocouple (type K). The measurement junction is inserted in the liquid and the reference junction is outside the vessel, whereas the temperature is $25$°C. The measured e.m.f. is $5.1878$ mV using a voltmeter inserted at the reference junction. The value of $E_{25,0}$ is $0.9918$ mV using the thermocouple table (see the handout given in the classroom). Try to find the liquid temperature.

Ans:

\[ E_{T,25} = 5.1878 \text{ mV}, E_{25,0} = 0.9918 \Rightarrow E_{T,0} = 6.1796 \text{ mV} \]

根據查表可得，$T=304$ °F
轉換為攝氏溫度為：$T = (304-32) \times (5/9) = 151.11$°C